

Instructions to candidates

Duration: 02 hours

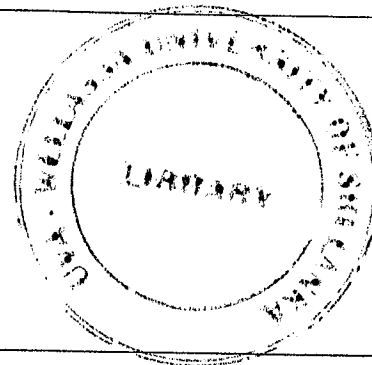
Number of questions: 4 Essay Questions

Mark allocation: 100 mark

Use standard symbols without definition.

Scientific calculators are allowed.

Answer all questions



1.

a. Find the Fourier transform of the function $f(x) = x^2$ in the interval $[-\pi, \pi]$. (20 mark)

b. Deduce that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}$. (5 mark)

2.

a. Find the Laplace transformation of the following functions.

i. $f(t) = t \sin at$ (5 mark)

ii. $f(t) = t^n e^{-at}$ (5 mark)

b. Find the inverse Laplace transformation of the following functions.

i. $F(s) = \frac{-10}{(s+1)^3}$ (5 mark)

ii. $F(s) = \frac{3s+1}{(s^2+s-12)}$ (5 mark)

iii. $F(s) = \frac{1}{(s+2)^2-9}$ (5 mark)

3.

- a. Using Laplace transformation, obtain an expression for the solution of the initial value problem given below,

$$ay'' + by' + cy = f(t); y(0) = y_0, y'(0) = y'_0. \quad (10 \text{ mark})$$

- b. Using Laplace transformation, solve

$$y'' + y = \sin 2t; y(0) = 2, y'(0) = 1. \quad (15 \text{ mark})$$

4. Compute the convolution of $a(t)$ and $b(t)$ (simplify your answer by showing all steps).

(25 mark)

$$a(t) = \begin{cases} 1; & -1 \leq t \leq 1 \\ 0; & \text{otherwise} \end{cases}$$

$$b(t) = \begin{cases} 1+t; & -1 \leq t \leq 0 \\ 1-t; & 0 < t \leq 1 \\ 0; & \text{otherwise} \end{cases}$$

